Modeling and Simulation of Levitating Ball by Electromagnet using Bond Graph

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Abstract—Magnetic Levitation has been a keen area of research, especially in the field of automotive where low losses due to friction and low energy consumption are important considerations. This paper offers the theoretical and experimental idea of the magnetic force control in the magnetic levitation system using the bond graph modeling approach considering the nonlinearities. The corresponding levitation apparatus comprises of an electromagnet, a ferroelectric material base, a steel ball and position sensors. The force generated by the electromagnet causes the levitation action over the ball by balancing the gravitational force exerted on the ball. The present work describes the linear and nonlinear model using signal flow graph and bond graph approach. Transfer function of the setup is obtained using signal flow graph and correspondingly the balancing action of the ball at its levitating position is performed using a PID controller. Lastly the overall system stability is validated using root locus approach.

Keywords—Magnetic levitation; Nonlinear model; Bond graph; PID Controller; experimental procedure

I. INTRODUCTION

Magnetic levitation has no contact between the moving object and fixed part and thus it is widely applicable in magnetic bearing, high speed ground transportation, vibration isolation, haptic interfaces, magnetic levitation applications such as precision motion control, vibration isolation, and haptic interfaces, magnetic levitation eliminates the friction and other dynamic nonlinearities such as hysteresis and cogging [7].

The particular magnetic levitation experimental setup uses as a magnetic ball suspension system which is used for levitating the steel (ferromagnetic material) ball in the air by the magneto motive force generated by the electromagnet [8]. The experimental setup consists of an electromagnet, a steel ball and a position sensor. The position of the ball is sensed by the embedded position sensor circuitry which is further used as a feedback signal to control the position of the steel ball [9-11]. The objective of the experiment is to design a controller that helps the steel ball to levitate at its equilibrium position. The ball position in the mechanical subsystem can be controlled by applying controlled voltage across the electromagnet terminals, thus the voltage applied across the electromagnet terminals provide an indirect control of the ball position. Such type of arrangement is utilized in making levitating globe. This paper is further organized as follows .The mathematical model of the same is enunciated in Section II, with a further elaboration of its bond graph model. The system behavior or dynamics is further analyzed with the help of the bond graph model which further aids in designing of the desired controller. Sections IV, V introduces about the experimental setup and experimental procedure. Further the paper concludes with a detailed discussion and validation of simulation and advantages of his work were almost zero amount of losses due to mechanical friction and the increased resolution and accuracy of the positioning device. They explained another advantage of a magnetic levitation system is that, it can operate as a rigid body rather than using jointed parts such as robots, which means that position errors do not compound and the dynamic behavior is simple to model. Magnetic levitation system has inherent instability thus it requires feedback control. P. Šuster et al. [6] designed the nonlinear maglev model into the Matlab Simulink and designed control algorithm together with simulation model of the Magnetic levitation and implemented into control structure with purpose of control on steady state of levitation setup. In the practical applications such as precision motion control, vibration isolation, and haptic interfaces, magnetic levitation eliminates the friction and other dynamic nonlinearities such as hysteresis and cogging [7].

experimental results of the maglev setup in Section VI and VII using root locus approach.

II. MATHEMATICAL MODELLING OF MAGLEV SETUP

The main objective is to design the control blocks and the transfer function of plant. The inductance of the coil changes with the change in position of the ball. The magnetic force on the ball is given by

\[ F_{\text{magnet}} = \frac{i^2}{2} \times \frac{dL}{dx} \]  

(1)

Where \( L \) is the total inductance of system, \( i \) is the current in the coil, and \( x \) is the position of the ball. The inductance of coil is composed of two components, the original inductance of coil and the additional inductance contributed by the ball at its equilibrium position. Let the variable \( x \) denote position of ball and \( x_0 \) denote the equilibrium position of the ball. Hence total inductance can be expressed as

\[ L = L_i + L_o \times \frac{x_0}{x} \]  

(2)

Where, \( L_i \) = inductance of the coil and \( L_o \) = incremental inductance of the ball

Writing the equation of ball motion we get,

\[ M \ddot{x} = Mg - F_{\text{magnet}} \]  

(3)

\[ F_{\text{magnet}} = C \times \frac{d^2x}{dx^2} \]  

(4)

Where, \( C = \frac{L_o \times x_0}{2} \)  

(5)

The corresponding equilibrium position obtained is,

\[ x_0 = \sqrt{\frac{C \times i^2}{M \times g}} \]  

(6)

It can be directly inferred from Eq. 3 that the present model is non-linear. Considering the linear approximation of the particular maglev setup the transfer function of the same is obtained, taking the Laplace of the Eq. 3.

\[ G_i(s) = \frac{X(s)}{I(s)} = \frac{-2C \times x_0}{M \times x_0^2 \times \left( s^2 - 2 \times C \times \frac{i^2}{M \times g} \right)} \]  

(7)

It can be observed that the input to this system is current \( i \) and output is the position of ball \( x \), thus \( G_i(s) \) provides us with transfer function of the electromechanical part of maglev setup. For the electrical part, we assume that the electromagnet coil is adequately modeled as a series resistor-inductor combination. Note that the inductor includes the steel ball and has the total inductance described previously. The voltage-current relationship for the coil with a simplifying assumption that the inherent inductance of the coil, \( L_i \) is much larger than the inductive contribution of the ball \( L_o \), and get the final equation as follows.

\[ V = iR + L_i \frac{di}{dt} \]  

(8)

After taking the Laplace of the Eq. 8 the transfer function of the electrical part is as follows.

\[ G_e(s) = \frac{I(s)}{V(s)} = \frac{\beta}{R + L_i s} \]  

(9)

Where, \( \beta \) is the gain of electrical system. Hence the total transfer function of the plant can be represented as \( G(s) = G_i(s) \times G_e(s) \) as shown in Fig. 1.

![Fig.1 Block diagram of open loop transfer function of Maglev setup](image1)

**Fig. 1** Block diagram of open loop transfer function of Maglev setup

![Fig.2 Block diagram of the closed loop controlled Magnetic levitation system](image2)

**Fig. 2** Block diagram of the closed loop controlled Magnetic levitation system.

Thus, the close loop transfer function is

\[ TF = \frac{G(s)}{1 + G(s)} \]  

(10)

Using this value of \( G(s) \) the root locus of the given system is plotted keeping negative feedback gain as unity. From the root locus plot as shown in Fig. 3, it can be inferred that the system is unstable for unity gain. Thus, the situation causes a demand of a controller to make the system stable.

![Fig 3.Root locus without PID](image3)

**Fig 3**.Root locus without PID

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### III. BOND GRAPH MODELING APPROACH OF LEVITATING OF BALL

A bond graph is a mechanism for studying dynamic systems. The Bond graph modeling approach was presented for the first time in the "Ports, Energy and Thermodynamic Systems" on April 24, 1959 at MIT by H.M Paynter. Bond graphs are used to map the flow of...
power from one part of a system to another. In the simplest form, a bond graph consists of subsystems linked together by lines representing power bonds. Bond graphs are not the only graphical means of system representation. There are other graphical ways of representing systems. Block diagrams and signal flow graphs are two such well-known techniques. Although they are similar to bond graphs, they are not quite the same. In both block diagrams and signal flow graphs, the links (arrows) used to link parts of a system carry only one type of information. In bond graphs (as discussed later) the half-arrow, or power bonds, carries power as an information which is further made of two variables, effort and flow [12].

A. Bond Graph Model of Levitating Ball with Core Loss

A solenoid transforms an electrical signal into mechanical movement. A typical solenoid consists of a coil wrapped around the fixed iron core. An increase in voltage \( V_{\text{in}} \), causes the current in the coil, thus increasing the core flux \( \phi \). The increasing flux further generates the magnetic force in the air gap which pulls the ball closer to the electromagnet. The corresponding schematic diagram of the maglev setup is as shown in Fig. 4.

\[
e = V_{\text{in}} - iR
\]

(11)

From Eq. 11 it can be concluded that, if amount of current changes the overall voltage will also be changed. This electromagnetic model is based on the change in magnetic flux through the moving ball and the coil. As we know, the amount of back emf is directly proportional to rate of change of flux and consecutively MMF is directly proportional to the current, thus we have

\[
e = N\phi
\]

(12)

Where, \( N \) is the number of turns in the coil and \( \phi \) is the amount of flux and \( M \) is magneto motive force. Here the effort is directly converted into flow, so the gyrator element is used in bond graph model. In the magneto mechanical model, MMF is produced due to the gap between moving and fixed core. Here three forces come in to picture, first the mechanical force due to the magnetic flux, secondly the MMF due to core gap and weight on the moving part. In the bond graph model mechanical force and MMF is connected with C field [13].

\[
M_g = \frac{x_0}{\mu_0 A}
\]

(14)

\[
F_g = \frac{\phi^2}{2\mu_0 A}
\]

(15)

\( M_g \) is magneto motive force, \( F_g \) is mechanical forces, \( x \) is length gap, \( \mu_0 \) is permeability of free space and \( A \) is gap area. Considering these forces the corresponding bond graph model produced is shown in Fig. 5.

\[
\begin{align*}
\phi &= \phi_0 + \Delta\phi \\
x &= x_0 + \Delta x \\
V &= V_0 + \Delta V
\end{align*}
\]

(16)

In view of small perturbation about the equilibrium the sources of effort changes as

\[
\begin{align*}
\frac{\phi'}{2\mu_0 A} &= \frac{\phi'}{\mu_0 A} + \frac{\Delta\phi}{\mu_0 A} \\
\frac{x}{\mu_0 A} &= \frac{x_0}{\mu_0 A} + x_0\Delta\phi + \frac{\Delta x}{\mu_0 A} \phi
\end{align*}
\]

(17)
In order to obtain the equilibrium distance, we assume that the core reluctance is negligible compared to the air gap reluctance. For the equilibrium distance $x_0$ assigned voltage is fixed and as a result the inductance does not contribute. In the similar way inertia and air resistance does not contribute in the equilibrium position. Thus we may write

$$N \frac{V}{R} = \frac{x_0}{\mu_0 A} \Delta$$

$$\frac{\Delta^2}{2 \mu_0 A} = mg.$$  (18)

The above relation asserts the equilibrium distance as

$$x_0 = \sqrt{\frac{N V \mu_0 A}{2 R mg}}.$$  (19)

With help of bondgraph model shown in Fig.7 signal flow graph drawn. Signal flow graph is a one method to find the transfer function of the sytem. Here authors shows the signal flow graph of the model in Fig7.

**Fig7. Signal flow graph of the bondgraph model of levitaing ball**

Signal flow graph is graphical representation of the dynamics of the control system and extensively used in design of the control system. Transfer functions obtain after solving the signal flow graph.

**B. Bond Graph Model of Levitating Ball negligible Flux Leakeage and Core Loss**

Here author assume that the air gap losses, core losses and air flux leakage flux is negligible then the final Bond graph model is given in the Fig. 8.

**Fig8. Bond graph model of magnetic levitating ball with negligible core loss**

$K_s$, is negative stiffness it is called as force- current factor, $K_i$ is force displacement factor and L is inductance.

$$K_s = \frac{L d_0}{x_0}$$  (30)

$$K_i = \frac{k_i d_0}{x_0}$$  (31)

$$L = \frac{N^2 A u i_0}{2 x_0}$$  (32)

Now with help of bond graph model signal flow graph is drawn. Again with the help of the signal flow graph of the corresponding model with negligible flux leakage and core loss as shown in Fig. 9, the transfer function is obtained.

![Fig.9 signal flow graph of the levitating ball](image)

After solving the signal flow graph we get the final transfer function of new bond graph model.

$$TF = \frac{p_1h_1}{s^3 + \frac{K_T}{L_m} + \frac{R}{mL_m}s + \frac{R}{mL_m}s^2}$$  \hspace{1cm} (33)

C. Bondgraph Model of Levitating Ball with PID Controller

Levitating ball is highly unstable system. Here our objective is to design a controller to control the position of the steel in the magnetic field. For stabilizing the position of the steel ball we used the PID controller. In the PID controller design three different forces are required to control the position error. Firstly an amount of force proportional to the positional error, secondly an amount of force proportional to the integral error and lastly is force proportional to the derivative error. Here author applied the PID control in the bond graph model of levitating ball without core loss and flux leakage. Model has been shown in the Fig. 10.

![Fig.10 Bond graph model of the levitating ball with PID controller](image)

IV. EXPERIMENTAL STUDY OF MAGLEV SETUP

This section enunciates the mechanical, electrical and its corresponding controller part of the entire maglev setup. Maglev unit consist of connection interface panel with a mechanical unit on which a coil is mounted. An infrared sensor attached to the mechanical unit and electrical unit measures the signal and sends it to the computer. Through the feedback technology mechanical and electrical unit connected to each other stabilizes the ball into the field of electromagnet. The experimental and basic structure of complete maglev setup is given in the Fig. 11.

![Fig.11. Magnetic Levitation Setup](image)

V. EXPERIMENTAL PROCEDURE

To proceed with the experiment, the following parameters of the levitation setup are determined which includes:

- Inductance of the coil $L=0.01\text{H}$
- Resistance of the coil $R=0.1\text{ohm}$
- Gain factor of the setup $\beta= 138$ (from experimental setup data)
- Diameter of ball $= 2.5\text{cm}$
- Mass of the ball $= 20\text{g}$ (steel alloy ball)

After finding these parameters the corresponding transfer function is evaluated with the help of mathematical modeling of levitating ball. With the help of root locus plot of the corresponding maglev system the corresponding desired values for proportional gain ($K_P$) and integral gain ($K_I$) and derivative gain ($K_D$) are determined. Then the Simulink model is interfaced with that of the data acquisition system to plot the graph between position and time with input voltage and stabilize the system.

VI. RESULT AND DISCUSSION

We balance the steel ball of equilibrium position by keeping gravitation and magnetic force equal. It can happen if there exists a system transfer function which is stable i.e. all the roots of the system should lie in the left hand side of the s-plane. But on plotting the root locus plot for the Eq. 24, it was observed that all the poles are lying in the right hand side thus making the system unstable as shown in the Fig. 3. Thus to obtain a stable system a PID controller was further incorporated. After doing the tuning with $K_P$, $K_I$, and $K_D$ parameters, the system got stable. As shown in Fig. 12, the root locus of the bondgraph model with PID controller is plotted. Similarly, the root locus of the experimental setup is also
obtained for both conditions i.e. without and with PID controllers embedded as shown in Fig. 13-14 respectively. Root locus is plot of the imaginary and real roots of the close loop transfer function with variation gain parameter.

In the Fig.3, 12, 13, 14 in the x axis real root and in y axis imaginary roots of transfer function.

The root locus for different gain values of controller has been observed. With the help of plot the gain of the system is obtained.

After taking root locus plot of experimental model and bondgraph model, a difference in measure of the gain values is observed. This situation arises because in case of experimental setup all the equation are linearized so it results in a second order transfer function whereas in the case of bondgraph model no such linearization assumption has been made thus resulting in a fourth order transfer function. Consecutively, the values of $R_{core}$ and $R_{air}$ are very less and its presence in the denominator term automatically increases the value of the gain drastically.

This difference is due to higher order of the order of transfer function, which is a resultant of consideration of flux leakage and core loss and control of the system is difficult even though for finding out gain value for stability is very much difficult but without consideration of the flux leakage and core loss as much simple For simulation, the model parameters considered are as follows:

- Diameter of ball = 2.5 cm
- Mass of the ball = 20 g (steel ball)
- $I$ = 4.8 amp (current in the coil)
- $N$ = 200 (number of turns of the coil)
- Inductance of the coil $L$ = 0.01 H
- Resistance of the coil $R$ = 0.1 ohm
- Gap area $A_g$ = 0.0002 m²

Other parameter like $R_{core}$ and $R_{air}$ are assumed to be very less and have been neglected.

With the help of these parameter author calculated the initial position for of ball as, $x_0 = 0.01109 m$.

As shown in Fig. 15, a position vs time graph is obtained for the maglev setup. From the respective plot it can be inferred that the system is highly unstable, thus requires a control strategy to obtain a stable system. To control the position of the steel ball tuning of the respective P-I-D controller gains are required. Thus, with the help of root locus plot the stability range of the gains is obtained and consequently the tuning procedure is performed. The tuning effects on simulation results are as shown in the Table 1.

Table 1: Effects of increasing parameters ($K_p$, $K_i$, $K_d$) on the maglev simulation

<table>
<thead>
<tr>
<th>Time(s)</th>
<th>Position (m)</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.6245</td>
<td>0.0306</td>
<td>0.05</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.2487</td>
<td>0.0612</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.8731</td>
<td>0.0918</td>
<td>0.15</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.4975</td>
<td>0.1224</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.1219</td>
<td>0.1530</td>
<td>0.25</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.7463</td>
<td>0.1836</td>
<td>0.3</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4.3707</td>
<td>0.2142</td>
<td>0.35</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4.9951</td>
<td>0.2448</td>
<td>0.4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
During the simulation of the maglev it is observed that the amount of proportional gain ($K_p$) and integral gain ($K_i$) are not affecting much in terms of stability but high value of $K_d$ is making the system unstable. Experimentally it has been found out that to make the system stable, the final range for $K_d$ should be less than 1 and $K_i$ should be more than 1 and range of $K_p$ should be more than 100. Thus for stabilizing the maglev ball we need at least the amount of $K_p=200$, $K_d=0.1$ and $K_i=1$. The position vs time plot for the corresponding tuned values of proportional, integral and derivative gains is as shown in Fig. 16. From Fig. 16 it can be inferred that a stable system is obtained.

**Fig. 16 Position vs Time Graph with PID controller of maglev setup**

<table>
<thead>
<tr>
<th>Gains</th>
<th>Rise time</th>
<th>Settling time</th>
<th>Steady State error</th>
<th>Stability</th>
<th>Overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>Increase</td>
<td>Increase</td>
<td>Increases</td>
<td>Not affected</td>
<td>Increase(min or)</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Decrease</td>
<td>Minor change</td>
<td>Increases</td>
<td>Improves if small</td>
<td>Decrease</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Minor affect</td>
<td>Decreases</td>
<td>Minor changes</td>
<td>Minor affect</td>
<td>Minor change</td>
</tr>
</tbody>
</table>

**VII. CONCLUSION**

The required parameters were found by performing experiment on the maglev setup. After applying the PID controller the steel ball was successfully levitated in a stable position in the magnetic field space. Value of the PID gain is determined by the root locus. In the bond graph modeling same work has done to replicate the experimental setup. System has been made stable with an addition of PID controller to actual unstable maglev setup. Non-linear behavior of the magnetic force and the dependence of total inductance of the maglev setup on the position of the steel ball challenged the stability of the system. Bond graph modeling of the setup added flexibility to the model and aided in calculation of $K_p$, $K_i$, and $K_d$ parameters of the controller and thus obtain a stable levitating position for the ball.

**REFERENCE**


